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The Fluid-Coupled Motion of Micro and Nanoscale Cantilevers

T Paramesh
Associate Professor, Department of Mechanical Engineering, SMEC Secunderabad.

Abstract: *An understanding of the fluid coupled dynamics of micro and nanotechnology has the potential to yield significant advances yet many open and interesting questions remain. As an important example we consider the coupling of two closely spaced cantilevers immersed in a viscous fluid subject to an external driving. While one cantilever is driven to oscillate, the adjacent cantilever is passive. This system is modeled as two simple harmonic oscillators in an array whose motion is coupled through the fluid. Using simplified geometries and the unsteady Stokes equations, an analytical expression is developed that describes the dynamics of the passive cantilever. Full numerical simulations of the fluid-solid interactions that include the precise geometries of interest are performed. The analytical expressions are compared with the numerical simulations to develop insight into the fluid-coupled dynamics over a range of experimentally relevant parameters including the cantilever separation and frequency based Reynolds number. In addition, a shaker-based actuation device is investigated in order to demonstrate its feasibility for use with micro and nanoscale systems.*

Keywords: *Cantilevers, Fluid Coupled, Nanoscale, Piezoshaker*

I. Introduction And Literature

The dynamics of an array of cantilevers in fluid can be exploited to make measurements with unprecedented sensitivities. Studying the dynamics of single molecules in their natural environment is one exciting possibility. In order to accomplish such a possibility, it is important to first understand the fluid-coupled dynamics of an array of cantilevers, which will benefit both biological and medical applications. As a result, new knowledge about these interactions needs to be developed.

To increase our understanding of the fluid-coupled dynamics of an array of cantilevers, an analytical expression is developed to predict the amplitude and phase of a fluid-coupled cantilever. The case of interest is an array of two cantilevers immersed in a viscous fluid separated by a known distance. One cantilever is driven to oscillate causing the surrounding fluid to move. The moving fluid then causes the neighboring cantilever to oscillate. Using both analytics and finite element numerical simulations we build an understanding of the fluid-coupled dynamics. The work presented here is summarized below.

1. Modeling the behavior of a single driven cantilever. The first part of this thesis considers a single cantilever immersed in a stationary, unbounded fluid. The cantilever is driven to oscillate sinusoidally. An analytical solution to the unsteady Stokes equations is used to provide a basic understanding of the fluid dynamics and the force that is exerted on the cantilever.
2. The fluid coupling between a single cantilever and a piezoshaker. We study the dynamics of a single cantilever immersed in fluid that is driven to oscillate by placing the cantilever and liquid on a piezoshaker. We determine the usefulness and efficiency of a piezoshaker device when the elastic objects are micro and nanoscale.
3. Analytical modeling of an array of cantilevers in fluid. We study the dynamics of an array of two cantilevers immersed in a fluid. One cantilever is driven externally and the adjacent cantilever oscillates because of the resulting fluid motion. An analytical expression is developed to predict the amplitude and phase of the fluid-coupled cantilever.
4. A numerical investigation of an array of cantilevers in fluid. We have performed finite element numerical simulations for the precise geometries of interest to validate our analytical predictions. The numerical simulations solve the complete fluid and solid equations and include the resulting fluid-solid interactions.

The overall contribution of this work is that we have shown that the piezoshaker method is promising for the actuation of micro scale systems that are immersed in a viscous fluid. Also, we have provided an analytical expression that describes the fluid-coupled dynamics of an array of cantilevers immersed in a viscous fluid, where one cantilever is driven externally and the adjacent cantilever oscillates because of the resulting fluid motion. The analytical expressions are validated using finite element numerical simulations for the precise geometries of interest and are anticipated to be of broad interest.

Analytical solution for an oscillating cantilever in fluid: Determining the fluid-coupled dynamics of an array of micro-scale cantilevers is challenging. In order to understand the fluid-coupled dynamics, it is appropriate to first study the dynamics of a single cantilever in fluid.
Flow field: Consider a long slender cantilever that is fixed at the base and free at the tip as shown in Figure. The cantilever is driven externally such that the time dependent displacement of the cantilever tip is given by

$$x_1(t) = A_0 \sin(\omega_d t)$$

Where A_0 is the amplitude and ω_d is the driving frequency.

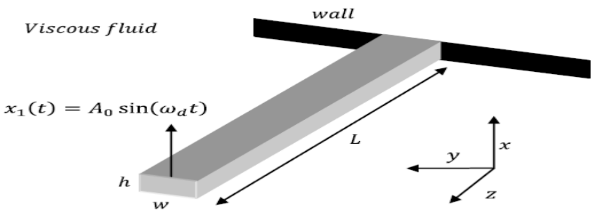


Figure - A cantilever of length L, width w, and height h immersed in a viscous fluid. The cantilever is driven with a displacement given by x1(t).

Table 1 - The beam geometry: length L, width w, thickness h. The resonant frequency in fluid and spring constant k. The beam is made of silicon with Young's modulus Ec = 1.74 1011 N / m2, and density c = 2320 kg / m3. The fluid is water with density f = 997 kg / m3, and dynamic viscosity µ = 8.59 10-4 kg / m s.

L	w	h	ω	k
197 μm	29 μm	2 μm	$158.5 \times 10^3\ rad\,s^{-1}$	$1.3\ N\,m^{-1}$

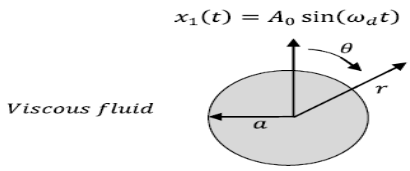


Figure - An infinite cylinder of radius a oscillating in a viscous fluid with a prescribed displacement x1(t). The coordinate r is measured from the center of the cylinder and is the clockwise angle measured with respect to the vertical.
The equations governing the fluid motion are the incompressible Navier-Stokes equations

$$\rho_f \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla}\vec{u}\right) = -\vec{\nabla}p + \mu\nabla^2\vec{u},$$

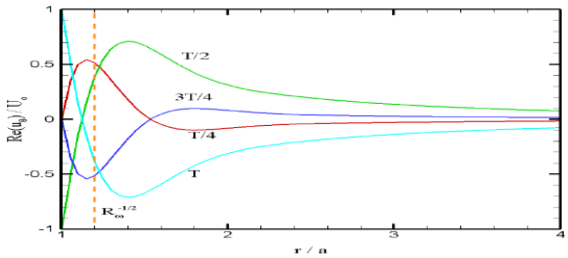
$$\vec{\nabla} \cdot \vec{u} = 0.$$

Where is the fluid velocity, is the pressure, is the fluid density, is time, and is dynamic viscosity. Eq. represents the conservation of momentum (where the body forces have been neglected). Eq. Represents the conservation of mass for an incompressible fluid.

$$R_\omega \frac{\partial \vec{u}}{\partial t} + R_u \vec{u} \cdot \vec{\nabla}\vec{u} = -\vec{\nabla}p + \nabla^2\vec{u}$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

Where and are the velocity and frequency based Reynolds numbers, respectively.



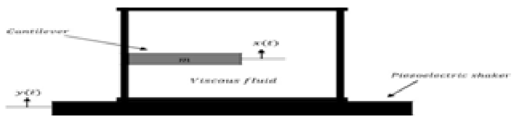
The flow field caused by an infinite cylinder with radius a oscillating in fluid. (Dashed Line) Normalized Stokes layer with radius a. (Left axis) Normalized real part of the flow field velocity with the maximum velocity U_0. (Bottom axis) Normalized radial distance.
Figure shows waves of momentum propagating in the radial direction and dissipating as time increases. The flow field is different when the cylinder is moving up (i.e. for time T) or down (i.e. for time T/2). It is clear that in the radial distance of the flow is in the viscous regime as expected.

The force acting on an oscillating cylinder in fluid
The force acting on an oscillating cylinder in fluid is derived in detail in Appendix B following the work of Stokes. The final expression is

$$F_f = -M' U_0 e^{i\omega t} i\omega \Gamma(R_\omega) = \gamma_f U_0 e^{i\omega t}$$

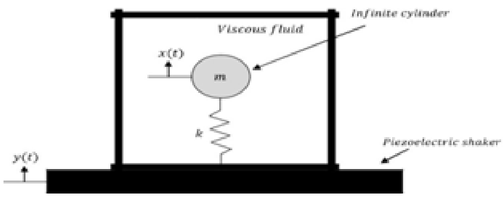
Where $M^I = \Pi a^2 \rho f$ is the mass per unit length of the fluid displaced by the cylinder and $\Gamma(R_w)$ is the hydrodynamic function given in Eq. F_f force that is proportional to the relative velocity $U_0 e^{i\omega t}$

The fluid coupled motion of a single cantilever and a piezoshaker: The piezoshaker method is an attractive method of actuation. For micro a nanoscale systems is small implying that the system is dominated by viscous forces. Determining how low of a Reynolds number it is still possible to see any significant oscillation of the beam is important for the usage of this method. This is of importance to our experimental collaborators because it is a readily available actuation mode for use with their current generation of NEMS devices. We emphasize that the rest of this thesis is valid for any actuation method and that this chapter stands alone as a study of one particularly interesting actuation mechanism.
Consider a cantilever with mass and spring constant that is immersed in a viscous fluid and placed on a piezoelectric actuator. Figure illustrates a piezoshaker where the displacement of the cantilever and the displacement of the actuator are represented by x(t) andy(t), respectively.

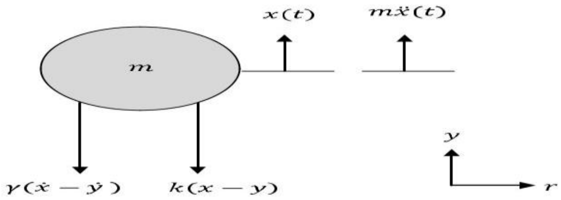


A cantilever of mass m and spring constant k immersed in a viscous fluid. The beam is excited by a piezoelectric shaker which has a prescribed harmonic displacement y(t). The displacement of the cantilever is given by x(t).

Again we consider the flow over a two dimensional cross section of the beam which is modeled as a cylinder (20) and is shown in Figure.



An infinite cylinder of mass per unit length m with spring constant k immersed in a viscous fluid. The cylinder is excited by a piezoelectric shaker which has a prescribed harmonic displacement y(t). The displacement of the cylinder is given by x(t).



Free body diagram of the forces acting on the mass m.
The forces acting on the mass are shown in Figure, where the viscous fluid damping is denoted by Y . The governing equation of motion is given by:

$$m\,\ddot{x} + \gamma\,(\dot{x} - \dot{y}) + \ k\,(x - y) = 0$$

where the piezoshaker displacement is given by,

$$y(t) = A\,\sin\,(\omega_d\ t)$$

Where A is the amplitude of the piezoshaker and ω_d is the driving frequency. Substituting y(t) in Eq. Yields,

$$m\,\ddot{x} + \gamma\dot{x} + \ kx = \gamma A\omega_d \cos(\omega_d t) + kA\sin(\omega_d t)$$

The fluid-coupled dynamics of an array of two cantilevers: We now study the fluid-coupled dynamics of an array of two micro-scale cantilevers immersed in fluid. We will develop an analytica
expression that governs the dynamics of the fluid-coupled cantilever. The expression determines the amplitude and phase of the fluid-coupled cantilever.
Consider two cantilevers immersed in fluid that are separated by a distance s, illustrated in Figure. The driven cantilever on the left has a displacement given by

$$x_1(t) = A_{0_1}\sin(\omega_d t)$$

Where A_{01} represents the amplitude of oscillation. The beam motion causes the fluid to move which causes the right cantilever beam to move

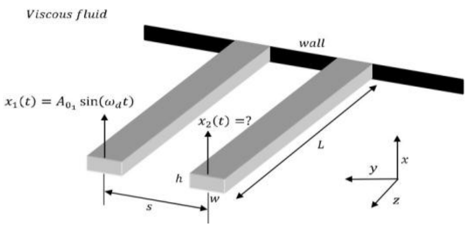
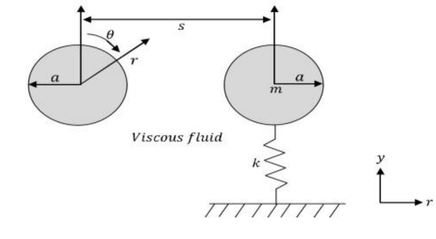


Figure: An array of two cantilevers of length L, width w, and height h that are separated by a distance s, which are immersed in a viscous fluid. The left cantilever has a displacement given by x1(t). The right cantilever beam has a displacement represented by x2(t), which is being moved by the fluid motion caused by the left cantilever.
As before, the cantilevers are modeled as infinite cylinders as shown in Figure.



Free body diagram for the right cylinder.

II. Results and Discussions:

Figure illustrates that the amplitude of oscillation decays quadratically as the normalized separation, s/a,increases, as expected from Eq. As R decreases the amplitude of oscillation for the right cylinder increases. This is because viscous forces increase with decreasing R_ω . Similarly, as R_ω decreases the Stokes length increases as $R_{\omega\,-}{}^{1/2}$.

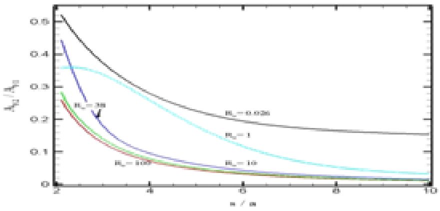


Figure shows that for R_ω<10

the phase of oscillation with respect to the driving cylinder is nearly constant and out of phase by ~ 2.8 radians (160o) as the normalized separation,s/a, increases. For an interesting phase variation occurs due to the fact that is increasing making the flow dominated by viscous effects. Therefore for the fluid coupling can be controlled by the varying normalized separation,

III. Conclusion

The dynamics of a single cantilever immersed in fluid that is driven to oscillate by means of a piezoshaker method was studied. It was shown that this method is promising for micro scale systems. Furthermore, the fluid-coupled dynamics of an array of two cantilevers immersed in fluid was considered. One cantilever was driven externally and the adjacent cantilever oscillated because of the resulting fluid motion. An analytical expression was developed for the amplitude and phase of a fluid-coupled cantilever. Full finite-element numerical simulations of the fluid-solid interactions for the precise geometries of interest were performed in order to validate the analytical expression. It was shown that the amplitude decays quadratically with increasing separation and that the fluid-coupled cantilever is nearly out of phase for any separation. The analytical expression provides important insight into analyzing and designing future micro and nanoscale technologies that are immersed in viscous fluids.

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